CHAPTER 2
Item Characteristic Curve Models

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Item Characteristic Curve Models

In the first chapter, the properties of the item characteristic curve were defined in terms of verbal descriptors. While this is useful to obtain an intuitive understanding of item characteristic curves, it lacks the precision and rigor needed by a theory. Consequently, in this chapter the reader will be introduced to three mathematical models for the item characteristic curve. These models provide a mathematical equation for the relation of the probability of correct response to ability. Each model employs one or more parameters whose numerical values define a particular item characteristic curve. Such mathematical models are needed if one is to develop a measurement theory that can be rigorously defined and is amenable to further growth. In addition, these models and their parameters provide a vehicle for communicating information about an item's technical properties. For each of the three models, the mathematical equation will be used to compute the probability of correct response at several ability levels. Then the graph of the corresponding item characteristic curve will be shown. The goal of the chapter is to have you develop a sense of how the numerical values of the item parameters for a given model relate to the shape of the item characteristic curve.

The Logistic Function

Under item response theory, the standard mathematical model for the item characteristic curve is the cumulative form of the logistic function. It defines a family of curves having the general shape of the item characteristic curves shown in the first chapter. The logistic function was first derived in 1844 and has been widely used in the biological sciences to model the growth of plants and animals from birth to maturity. It was first used as a model for the item characteristic curve in the late 1950s and, because of its simplicity, has become the preferred model. The equation for the two-parameter logistic model is given in equation 2-1 below.

$$P(\theta) = \frac{1}{1+e^{-L}} = \frac{1}{1+e^{-a(\theta-b)}}$$

[2-1]

where: **e** is the constant 2.718

b is the difficulty parameter

a is the discrimination parameter[1]

L = $a(\theta$ - b) is the logistic deviate (logit) and

θ is an ability level.

The difficulty parameter, denoted by b, is defined as the point on the ability scale at which the probability of correct response to the item is .5. The theoretical range of the values of this parameter is - 4 < = b < =   + 4. However, typical values have the range -3 < = b < = +3.

Due to the S shape of the item characteristic curve, the slope of the curve changes as a function of the ability level and reaches a maximum value when the ability level equals the item's difficulty. Because of this, the discrimination parameter does not represent the general slope of the item characteristic curve as was indicated in Chapter 1. The technical definition of the discrimination parameter is beyond the level of this book. However, a usable definition is that this parameter is proportional to the slope of the item characteristic curve at θ = b. The actual slope at θ = b is a/4, but considering a to be the slope at b is an acceptable approximation that makes interpretation of the parameter easier in practice. The theoretical range of the values of this parameter is - 4< = a < = + 4, but the usual range seen in practice is -2.80 to +2.80.

Computational Example

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1. In much of the item response literature, the parameter *a* is reported as a normal ogive model value that is then multiplied by 1.70 to obtain the corresponding logistic value. This is done to make the two-parameter logistic ogive similar to the normal ogive. However, this was not done in this book because it introduces two frames of reference for interpreting the numerical values of the discrimination parameter. All item parameters in this book and the associated computer programs are interpreted in terms of the logistic function. Thus, the reported values would be divided by 1.70 to obtain the corresponding normal ogive values.

To illustrate how the two-parameter model is used to compute the points on an item characteristic curve, consider the following example problem. The values of the item parameters are:

b = 1.0 is the item difficulty.
a = .5 is the item discrimination.

The illustrative computation is performed at the ability level θ = -3.0.

The first term to be computed is the logistic deviate (logit), L, where:

L = a (θ - b).

Substituting the appropriate values yields:

L = .5 (-3.0 - 1.0) = -2.0.

The next term computed is **e** (2.718) raised to the power -L. If you have a pocket calculator that can compute **e**^{x} you can verify this calculation. Substituting yields:

EXP (-L) = EXP (2.0) = 7.389, where EXP represents **e**.

Now the denominator of equation 2-1 can be computed as:

1 + EXP (-L) = 1 + 7.389 = 8.389.

Finally, the value of P(θ) is:

P(θ) = 1/(1 + EXP (-L)) = 1/8.389 = .12.

Thus, at an ability level (T) of -3.0, the probability of responding correctly to this item is .12.

From the above, it can be seen that computing the probability of correct response at a given ability level is very easy using the logistic model. Table 2-1 shows the calculations for this item at seven ability levels evenly spaced over

the range of abilities from -3 to +3. You should perform the computations at
several of these ability levels to become familiar with the procedure.

TWO-PARAMETER MODEL

$$P = 1/(1 + EXP(-A(T - B)))$$
$$P = 1/(1 + EXP(- (.5) (T - (1))))$$

Ability	Logit	EXP(-L)	1 = EXP(-L)	P
-3	-2	7.389	8.389	.12
-2	-1.5	4.482	5.482	.18
-1	-1	2.718	3.718	.27
0	-.5	1.649	2.649	.38
1	0	1	2	.5
2	.5	.607	1.607	.62
3	1	.368	1.368	.73

Table 2-1. Item characteristic curve calculations under a
two-parameter model, *b* = 1.0, *a* = .5

The item characteristic curve for the item of Table 2-1 is shown below. The
vertical arrow corresponds to the value of the item difficulty.

where: b is the difficulty parameter and θ is the ability level.

It should be noted that a discrimination parameter was used in equation 2-2, but because it always has a value of 1.0, it usually is not shown in the formula.

Computational Example

Again the illustrative computations for the model will be done for the single ability level -3.0. The value of the item difficulty parameter is:

$b = 1.0$.

The first term computed is the logit, L, where:

$L = a\,(\theta - b)$

Substituting the appropriate values yields:

$L = 1.0\,(-3.0 - 1.0) = -4.0$

Next, the **e** to the x term is computed, giving:

EXP(-L) = 54.598

The denominator of equation 2-2 can be computed as:

1 + EXP(-L) = 1.0 + 54.598 = 55.598

Finally, the value of P(θ) can be obtained and is:

P(0) = 1/(1 + EXP(-L)) = 1/55.598 = .02

Thus, at an ability level of -3.0, the probability of responding correctly to this item is .02. Table 2-2 shows the calculations for seven ability levels. You should perform the computations at several other ability levels to become familiar with the model and the procedure.

The item characteristic curve corresponding to the item in Table 2-2 is shown below.

ONE-PARAMETER MODEL

$$P = 1/(1 + EXP(-1(T - B)))$$
$$P = 1/(1 + EXP (-1 (T - (1))))$$

Ability	Logit	EXP(-L)	1 = EXP(-L)	P
-3	-4	54.598	55.598	.02
-2	-3	20.086	21.086	.05
-1	-2	7.389	8.389	.12
0	-1	2.718	3.718	.27
1	0	1	2	.5
2	1	.368	1.368	.73
3	2	.135	1.135	.88

Table 2-2. Calculations for the one-parameter model, *b* = 1.0
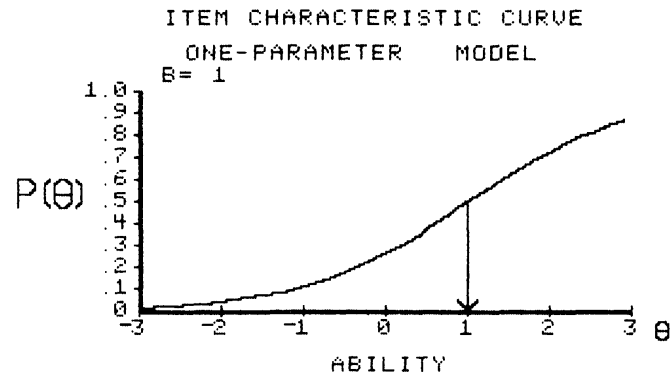

FIGURE 2-2. Item characteristic curve for a
one-parameter model with *b* = 1.0

The Three-Parameter Model

One of the facts of life in testing is that examinees will get items correct by guessing. Thus, the probability of correct response includes a small component that is due to guessing. Neither of the two previous item characteristic curve models took the guessing phenomenon into consideration. Birnbaum (1968) modified the two-parameter logistic model to include a parameter that represents the contribution of guessing to the probability of correct response. Unfortunately, in so doing, some of the nice mathematical properties of the logistic function were lost. Nevertheless the resulting model has become known as the three-parameter logistic model, even though it technically is no longer a logistic model. The equation for the three-parameter model is:

$$P(\theta) = c + (1 - c)\frac{1}{1 + e^{-a(\theta - b)}} \qquad \text{[2-3]}$$

where: b is the difficulty parameter
a is the discrimination parameter
c is the guessing parameter and
θ is the ability level

The parameter c is the probability of getting the item correct by guessing alone. It is important to note that by definition, the value of c does not vary as a function of the ability level. Thus, the lowest and highest ability examinees have the same probability of getting the item correct by guessing. The parameter c has a theoretical range of $0 <= c <= 1.0$, but in practice, values above .35 are not considered acceptable, hence the range $\theta \# c \# .35$ is used here.

A side effect of using the guessing parameter c is that the definition of the difficulty parameter is changed. Under the previous two models, b was the point on the ability scale at which the probability of correct response was .5. But now, the lower limit of the item characteristic curve is the value of c rather than zero. The result is that the item difficulty parameter is the point on the ability scale where:

$$P(\theta) = c + (1 - c)\ (.5)$$

$$= (1 + c)/2$$

This probability is halfway between the value of c and 1.0. What has happened here is that the parameter c has defined a floor to the lowest value of the probability of correct response. Thus, the difficulty parameter defines the point on the ability scale where the probability of correct response is halfway between this floor and 1.0.

The discrimination parameter a can still be interpreted as being proportional to the slope of the item characteristic curve at the point $\theta = b$. However, under the three-parameter model, the slope of the item characteristic curve at $\theta = b$ is actually $a\ (1 - c)/4$.

While these changes in the definitions of parameters b and a seem slight, they are important when interpreting the results of test analyses.

Computational Example

The probability of correct response to an item under the three-parameter model will be shown for the following item parameter values:

$b = 1.5$, $a = 1.3$, $c = .2$

at an ability level of $\theta = -3.0$.

The logit is:

$L = a\ (\theta - b) = 1.3\ (-3.0 - 1.5) = -5.85$

The **e** to the x term is:

$EXP(-L) = EXP(5.85) = 347.234$

The next term of interest is:

$1 + EXP(-L) = 1.0 + 347.234 = 348.234$

and then,

$1/(1 + EXP(-L)) = 1/348.234 = .0029$

Up to this point, the computations are exactly the same as those for a two-parameter model with $b = 1.5$ and $a = 1.3$. But now the guessing parameter enters the picture. From equation 2-3 we have:

$$P(\theta) = c + (1 - c)\,(.0029) \text{ and,}$$

$c = .2$ so that:

$$P(\theta) = .2 + (1.0 - .2)\,(.0029)$$
$$= .2 + (.80)(.0029)$$
$$= .2 + (.0023)$$
$$= .2023$$

Thus, at an ability level of -3.0, the probability of responding correctly to this item is .2023. Table 2-3 shows the calculations at seven ability levels. Again, you are urged to perform the above calculations at several other ability levels to become familiar with the model and the procedures.
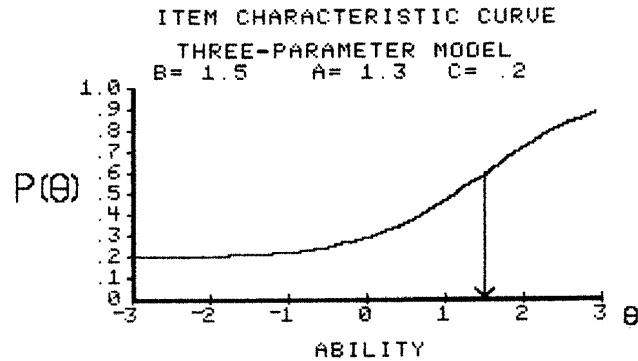
THREE-PARAMETER MODEL

$$P = C + (1 - C)\,(1/(1 + EXP(-A\,(T - B))))$$
$$P = .2 + (1 - .2)(1/(1 + EXP\,(-\,(1.3)\,(T - (1.5))))))$$

Ability	Logit	EXP(-L)	1 + EXP(-L)	P
-3	-5.85	347.234	348.234	.2
-2	-4.55	94.632	95.632	.21
-1	-3.25	25.79	26.79	.23
0	-1.95	7.029	8.029	.3
1	-.65	1.916	2.916	.47
2	.65	.522	1.522	.73
3	1.95	.142	1.142	.9

Table 2-3. Calculations for the three-parameter model,
$b = 1.5$, $a = 1.3$, $c = .2$

The corresponding item characteristic curve is shown below.

```
          ITEM CHARACTERISTIC CURVE
           THREE-PARAMETER MODEL
           B= 1.5      A= 1.3    C= .2
     1.0
      .9
      .8
      .7
      .6
P(θ) .5
      .4
      .3
      .2
      .1
      0
        -3    -2    -1    0     1     2    3  θ
                  ABILITY
```

FIGURE 2-3. Item characteristic curve for a three-parameter model with *b* = 1.5, *a* = .3, *c* = .2
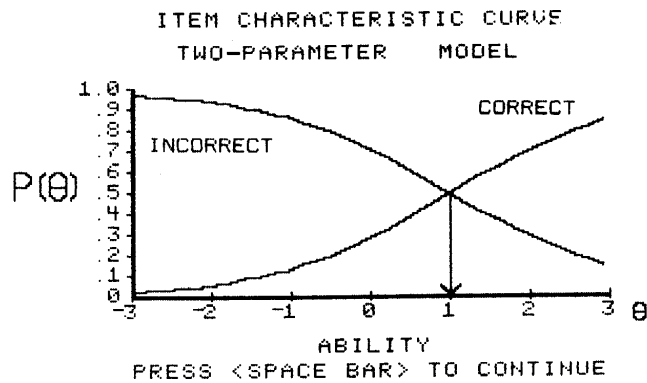
Negative Discrimination

While most test items will discriminate in a positive manner (i.e., the probability of correct response increases as the ability level increases), some items have negative discrimination. In such items, the probability of correct response decreases as the ability level increases from low to high. Figure 2-4 depicts such an item.

ITEM CHARACTERISTIC CURVE

TWO-PARAMETER   MODEL
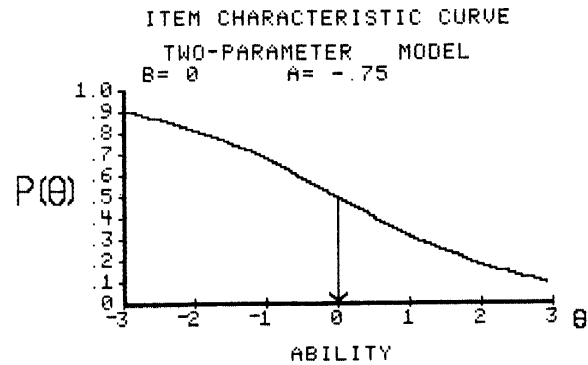
B= 0   A= -.75

FIGURE 2-4. An item with negative discrimination
under a two-parameter model with *b* = 0, *a* = -.75

Items with negative discrimination occur in two ways. First, the incorrect
response to a two-choice item will always have a negative discrimination
parameter if the correct response has a positive value. Second, sometimes the
correct response to an item will yield a negative discrimination index. This tells
you that something is wrong with the item: Either it is poorly written or there
is some misinformation prevalent among the high-ability students. In any case,
it is a warning that the item needs some attention. For most of the item
response theory topics of interest, the value of the discrimination parameter
will be positive. Figure 2-5 shows the item characteristic curves for the correct
and incorrect responses to a binary item.

```
            ITEM  CHARACTERISTIC  CURVE
              TWO-PARAMETER     MODEL
      1.0
       .9                          CORRECT
       .8
       .7   INCORRECT
       .6
P(θ)   .5
       .4
       .3
       .2
       .1
       0
        -3     -2    -1    0    1    2    3  θ
                     ABILITY
           PRESS <SPACE BAR> TO CONTINUE
```

FIGURE 2-5. Item characteristic curves for the correct
(*b* = 1.0, *a* = .9) and incorrect responses
(*b* = 1.0, *a* = -.9) to a binary item

It should be noted that the two item characteristic curves have the same value
for the difficulty parameter *(b = 1.0)* and the discrimination parameters have
the same absolute value. However, they have opposite signs, with the correct
response being positive and the incorrect response being negative.

Guidelines for Interpreting Item Parameter Values

In Chapter 1, verbal labels were used to describe the technical properties of an
item characteristic curve. Now the curves can be described via parameters
whose numerical values have intrinsic meaning. However, one needs some
means of interpreting the numerical values of the item parameters and
conveying this interpretation to a non-technical audience. The verbal labels
used to describe an item's discrimination can be related to ranges of values of
the parameter as follows:

Verbal label	Range of values
none	0
very low	.01 - .34
Low	.35 - .64
moderate	.65 - 1.34
High	1.35 - 1.69
Very high	> 1.70
Perfect	+ infinity

Table 2-4. Labels for item discrimination parameter values

These relations hold when one interprets the values of the discrimination parameter under a logistic model for the item characteristic curve. If the reader wants to interpret the discrimination parameter under a normal ogive model, divide these values by 1.7.

Establishing an equivalent table for the values of the item difficulty parameter poses some problems. The terms *easy* and *hard* used in Chapter 1 are relative terms that depend upon some frame of reference. As discussed above, the drawback of item difficulty, as defined under classical test theory, was that it was defined relative to a group of examinees. Thus, the same item could be easy for one group and hard for another group. Under item response theory, an item's difficulty is a point on the ability scale where the probability of correct response is .5 for one- and two-parameter models and $(1 + c_i)/2$ for a three-parameter model. Because of this, the verbal labels used in Chapter 1 have meaning only with respect to the midpoint of the ability scale. The proper way to interpret a numerical value of the item difficulty parameter is in terms of where the item functions on the ability scale. The discrimination parameter can be used to add meaning to this interpretation. The slope of the item characteristic curve is at a maximum at an ability level corresponding to the item difficulty. Thus, the item is doing its best in distinguishing between examinees in the neighborhood of this ability level. Because of this, one can speak of the item functioning at this ability level. For example, an item whose

difficulty is -1 functions among the lower ability examinees. A value of +1 denotes an item that functions among higher ability examinees. Again, the underlying concept is that the item difficulty is a location parameter.

Under a three-parameter model, the numerical value of the guessing parameter c is interpreted directly since it is a probability. For example, $c = .12$ simply means that at all ability levels, the probability of getting the item correct by guessing alone is .12.

Computer Session for Chapter 2

The purpose of this session is to enable you to develop a sense of the dependence of the shape of the item characteristic curve upon the model and the numerical values of its parameters. You will be able to set the values of the parameters under each of the three models and view the corresponding item characteristic curve on the screen. Choosing the values becomes a function of what kind of an item characteristic curve one is trying to define. Conversely, given a set of numerical values for an item characteristic curve such as provided by a test analysis, you should be able to visualize the form of the item characteristic curve from these values. Such visualization is necessary to properly interpret the technical properties of the item. After doing the exercises and a bit of exploration, you should be able to visualize the form of the item characteristic curve for any model and set of parameter values.

Procedures for an Example Case

a.   Follow the start-up procedures described in the Introduction.

b.   Use the mouse to highlight ITEM PARAMETERS, then click on [CONTINUE].

c.   Read the explanatory screen and click on [CONTINUE]. The
      SELECT ITEM CHARACTERISTIC CURVE MODEL screen
      will appear.

d.   Use the mouse to click on TWO PARAMETER, then click on
      [CONTINUE]. The ITEM PARAMETER screen will appear.

e.   Click on [ENTER PARAMETER VALUES], then set the values of
      the item parameters to $a = 1.7$ and $b = -1.0$. Select YES for
      VALUE(S) OK?

f.   The computer will display the table of computations. Study the
      table for a few minutes to see the relation between the probability
      of correct response and the ability scores.

g.   Click on [CONTINUE]. The item characteristic curve will be
      displayed on the screen.

h.   This item functions at an ability level of -1, and the curve is quite
      steep at that ability level. Notice that the curve is nearly flat above
      an ability of 1.0. In technical terms, it is becoming asymptotic to a
      value of $P(\theta) = 1.0$.

i.   Click on [CONTINUE]. The SELECT OPTIONS screen will
      appear.

j.   At this juncture, the example case is completed. Respond to the
      question ANOTHER ITEM? by clicking on the YES button. Then
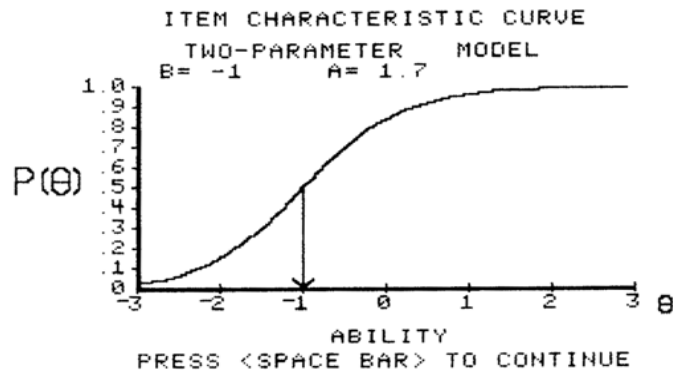   proceed to Exercise 1.

TWO-PARAMETER MODEL

$$P = 1/(1 + EXP(-A\ (T-B)))$$
$$P = 1/(1 + EXP\ (-\ (1.7)\ (T-(-1))))$$

Ability	Logit	EXP(-L)	1 + EXP(-L)	P
-3	-3.4	29.964	30.964	.03
-2	-1.7	5.474	6.474	.15
-1	0	1	2	.5
0	1.7	.183	1.183	.85
1	3.4	.033	1.033	.97
2	5.1	6E-03	1.006	.99
3	6.8	1E-03	1.001	1

Table 2-5. Calculations for an item with *b* = -1.0, *a* = 1.7
under a two-parameter model

FIGURE 2-6. Item characteristic curve for
b = -1.0, *a* = 1.7

Exercises

Exercise 1

This exercise uses the Rasch model to illustrate how the difficulty parameter locates an item along the ability scale.

a.   Respond to the question SAME MODEL? by clicking on the NO button.

b.   Respond to the question SHOW CALCULATIONS? by clicking on the YES button.

c.   Respond to the question PLOT ON SAME GRAPH? by clicking on the NO button.

d.   Respond to the question SELECTIONS OK? by clicking on the YES button. The SELECT ITEM CHARACTERISTIC CURVE MODEL screen will appear.

e.   Click on RASCH, then click on [CONTINUE]. The ITEM PARAMETER VALUES screen will appear.

f.   Set the value of the difficulty parameter to $b = -2.0$. and respond to VALUE(S) OK by clicking on the YES button.

g.   The computer will display the table of computations. Study the table for a few minutes to see the relation between the probability of correct response and the ability scores.

h.   Click on [CONTINUE]. The item characteristic curve will be displayed on the screen.

i.   This item will function at an ability level of -2.0, and the curve will be moderately steep at that ability level. Study the plot, then click on [CONTINUE]. The SELECT OPTIONS screen will appear.

j.   Next, we want to put another item characteristic curve on the same graph. Respond to the question ANOTHER ITEM? by clicking on the YES button.

k.   Respond to the question SAME MODEL? by clicking on the YES button.

l.   Respond to the question SHOW CALCULATIONS? by clicking on the YES button.

m.   Respond to the question PLOT ON THE SAME GRAPH? by clicking on the YES button.

n.   Now repeat Steps d through h, but set the item difficulty to a value of $b = 0.0$.

o.   This will place a second curve on the graph.

p.   Now repeat Steps i through n, using the value $b = 2.0$ for the item difficulty.

q.   Now there will be three item characteristic curves on the screen. If you place a straight edge across the screen at $P(\theta) = .5$, it will intersect these curves at the ability levels defined by their difficulties. In the current example, the difficulties are evenly spaced along the ability scale.

r.   Click on [CONTINUE]. The SELECT OPTIONS screen will appear.

s.   At this juncture, Exercise 1 is completed. Respond to the question ANOTHER ITEM? by clicking on the YES button.

Exercise 2

This exercise uses the two-parameter model to illustrate the joint effect of item difficulty and discrimination upon the shape of item characteristic curve.

a.   Respond to the question SAME MODEL? by clicking on the NO button.

b.   Respond to the question SHOW CALCULATIONS? by clicking on the YES button.

c.   Respond to the question PLOT ON SAME GRAPH? by clicking on the NO button.

d.   Respond to the question SELECTIONS OK? by clicking on the YES button. The SELECT ITEM CHARACTERISTIC CURVE MODEL screen will appear.

e.   Click on TWO PARAMETER, then click on [CONTINUE]. The ITEM PARAMETER VALUES screen will appear.

f.   Set the value of the item parameters to $a = 1.0$, $b = -2.0$. and respond to VALUE(S) OK? by clicking on the YES button.

g.   The computer will display the table of computations. Study the table for a few minutes to see the relation between the probability of correct response and the ability scores.

h.   Click on [CONTINUE]. The item characteristic curve will be displayed on the screen.

i.   The item characteristic curve is located in the low-ability end of the scale and is moderately steep.

j.   Next, we want to put another item characteristic curve on the same graph. Respond to the question ANOTHER ITEM? by clicking on the YES button.

k.   Respond to the question SAME MODEL? by clicking on the YES button.

l.   Respond to the question SHOW CALCULATIONS? by clicking on the YES button.

m.   Respond to the question PLOT ON THE SAME GRAPH? by clicking on the YES button.

n.   Now repeat Steps d through h, but set the item parameters to $a = 1.5$, $b = 0.0$.

o.   This will place a second curve on the graph.

p.   Now repeat Steps j through m, using the values $a = .5$, $b = 2.0$.

q.   You should now have three item characteristic curves displayed on the same graph. It should be clear that the value of b locates the item on the ability scale and that a defines the slope. However, in the current example, the curves cross because the values of a are different for each item.

r.   Click on [CONTINUE]. The SELECT OPTIONS screen will appear.

s.   At this juncture, Exercise 2 is completed.

Exercise 3

This exercise illustrates the joint effect of the parameter values under the three-parameter model.

a.   Respond to the question ANOTHER ITEM? by clicking on the YES button.

b.   Respond to the question SAME MODEL? by clicking on the NO

button.

c.   Respond to the question SHOW CALCULATIONS? by clicking on
       the YES button.

d.   Respond to the question PLOT ON SAME GRAPH? by clicking
       on the NO button.

e.   Respond to the question SELECTIONS OK? by clicking on the
       YES button. The SELECT ITEM CHARACTERISTIC CURVE
       MODEL screen will appear.

f.   Click on THREE PARAMETER, then click on [CONTINUE].
       The ITEM PARAMETER VALUES screen will appear.

g.   Set the value of the item parameters to $a = 1.0$, $b = -2.0$, and $c =$
       .10, and respond to VALUE(S) OK? by clicking on the YES
       button.

h.   The computer will display the table of computations. Study the
       table for a few minutes to see the relation between the probability
       of correct response and the ability scores.

i.   Click on [CONTINUE]. The item characteristic curve will be
       displayed on the screen. After studying the curve, click on
       [CONTINUE]. The SELECT OPTIONS SCREEN will appear.

j.   Respond to the question ANOTHER ITEM? by clicking on the
       YES button.

k.   Respond to the question SAME MODEL? by clicking on the YES
       button.

l.   Respond to the question SHOW CALCULATIONS? by clicking on
       the YES button.

m.   Respond to the question PLOT ON SAME GRAPH? by clicking
      on the YES button.

n.   Respond to the question SELECTIONS OK? by clicking on the
      YES button. The SELECT ITEM CHARACTERISTIC CURVE
      MODEL screen will appear.

o.   Repeat steps f through h, using item parameter values of $b = 0.0$, a
      $= 1.5$, $c = .20$.

p.   Repeat steps b through h, using item parameter values of $b = 2.0$, a
      $= .5$, $c = .30$.

q.   At this point, you should have three item characteristic curves
      displayed on the graph. Again the value of b locates the items along
      the ability scale, but the ability level at which $P(\theta) = .5$ does not
      correspond to the value of b but is slightly lower. Recall that under
      the three-parameter model, b is the point on the ability scale where
      the probability of correct response is $(1 + c)/2$ rather than .5. The
      slopes of the curves at b reflect the values of a. The lower tails of
      the three curves approach their values of c at the lowest levels of
      ability. However, this is not apparent for the curve with $b = -2.0$
      because the values of $P(\theta)$ are still rather large at $\theta = -3.0$.

Exercise 4

a.   For each model:

      (1) Select a set of parameter values.

      (2) Show at least one calculation table.

      (3) Predict what the shape of the item characteristic curve will look
           like. It can be helpful to make a sketch of the item characteristic
           curve before the computer shows it on the screen.

(4) Obtain the display of the curve. (It may help to overlay a few curves to get a feeling for the relative effects of changing parameter values.)

b. Repeat this process until you know what kind of item characteristic curve will result from a set of numerical values of the item parameters under each of the models.

Things To Notice

1. Under the one-parameter model, the slope is always the same; only the location of the item changes.

2. Under the two- and three-parameter models, the value of a must become quite large (>1.7) before the curve is very steep.

3. Under Rasch and two-parameter models, a large positive value of b results in a lower tail of the curve that approaches zero. But under the three-parameter model, the lower tail approaches the value of c.

4. Under a three-parameter model, the value of c is not apparent when $b < 0$ and a < 1.0. However, if a wider range of values of ability were used, the lower tail would approach the value of c.

5. Under all models, curves with a negative value of a are the mirror image of curves with the same values of the remaining parameters and a positive value of a.

6. When $b = -3.0$, only the upper half of the item characteristic curve appears on the graph. When $b = +3.0$, only the lower half of the curve appears on the graph.

7. The slope of the item characteristic curve is the steepest at the ability level corresponding to the item difficulty. Thus, the difficulty parameter b locates the point on the ability scale where the item functions best.

8. Under the Rasch and two-parameter models, the item difficulty defines the point on the ability scale where the probability of correct response for persons of that ability is .5. Under a three-parameter model, the item of difficulty defines the point on the ability scale where the probability of correct response is halfway between the value of the parameter c and 1.0. Only when $c = 0$ are these two definitions equivalent.